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Ampacity Studies of Riverside Jct x Strachan TS Cable Circuits

Report No. 88-86-K

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ABSTRACT

The results of computations of cable ampacities based on field measurements of thermal resistivities and the application of the classical IEC/Neher-McGrath techniques are reported. This report also introduces a probabilistic method for thermal analysis of power cables in the system planning process. An advanced finite element method in conjunction with the Monte Carlo simulation is applied for calculating the temperature rise of power cables with provision for statistical variations of various soil, boundary and loading conditions. The probability distribution of cable operating temperature is obtained and used in performing cost-benefit analysis of delaying new cable installations.

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G.J. Anders

1.O INTRODUCTION

As a result of the discussions at the Underground Power Transmission Research Panel meeting on September 23, 1985, a Task Force was formed to perform a study and measurements to review ampacities of twelve 115 kV cables between Riverside Jct and Strachan TS. If the ampacities of these cables can be increased, significant savings can be achieved by deferring the need for new circuits. If, on the other hand, the original ampacities were found to be to optimistic, system operators will have to take remedial action to prevent possible cable damage. The analysis and measurement methods proposed by the Task Force were to be eventually applied to other cable installations in the province.

The cables under consideration are hollow core, low pressure oil filled cables (LPOF) with copper segmental construction, lead sheath and rubber bitumen sandwich protection (see Figure 1). The cables have the following specifications:

conductor size	1,250 MCM
core diameter	d_c = 0.5"
conductor diameter	d = 1.255"
insulation thickness	T = 0.43"
diameter over insulation	D_c = 2.15"
sheath thickness	t_s = 0.115"
diameter over the sheath	D_s = 2.39"
jacket thickness	t_j = 0.15"
overall cable diameter	D_φ = 2.69"

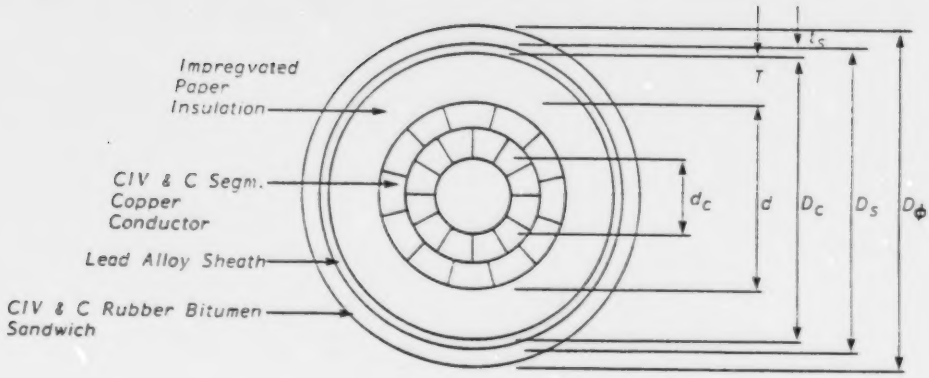


Figure 1. Cable cross section

The cable arrangements for two critical cross sections are shown in Figure 2.

Measurements of soil thermal resistivity (28 measurements altogether) revealed very high thermal resistivity values of the sand backfill in the vicinity of mature trees (up to 4.0 °K•m/W as compared with the design value of 0.9 °K•m/W). This called for the revision of the cable ratings for power system operation and resulted in a review of planning objectives with regard to these cables.

Two types of studies were performed to assess the ratings of these cables. In the first case, the FIECAG program was used with the measured values of thermal resistivities. In the second case, a probabilistic analysis of cable temperatures was performed. The techniques applied in the second case and the results of both studies are presented in this report. For background information on FIECAG computer program, the reader is referred to References 1 and 6.

2.0 FIECAG STUDIES

The measured thermal resistivities of the sand backfill and native soil in the vicinity of the trees and in the area away from trees are summarized in Table 1.

TABLE 1

Thermal Resistivities in FIECAG Studies

	Cases with trees		
	Single trench	Two trenches	Case without trees
Backfill	4.0	4.0	2.0
Native soil	2.0	1.5	1.0

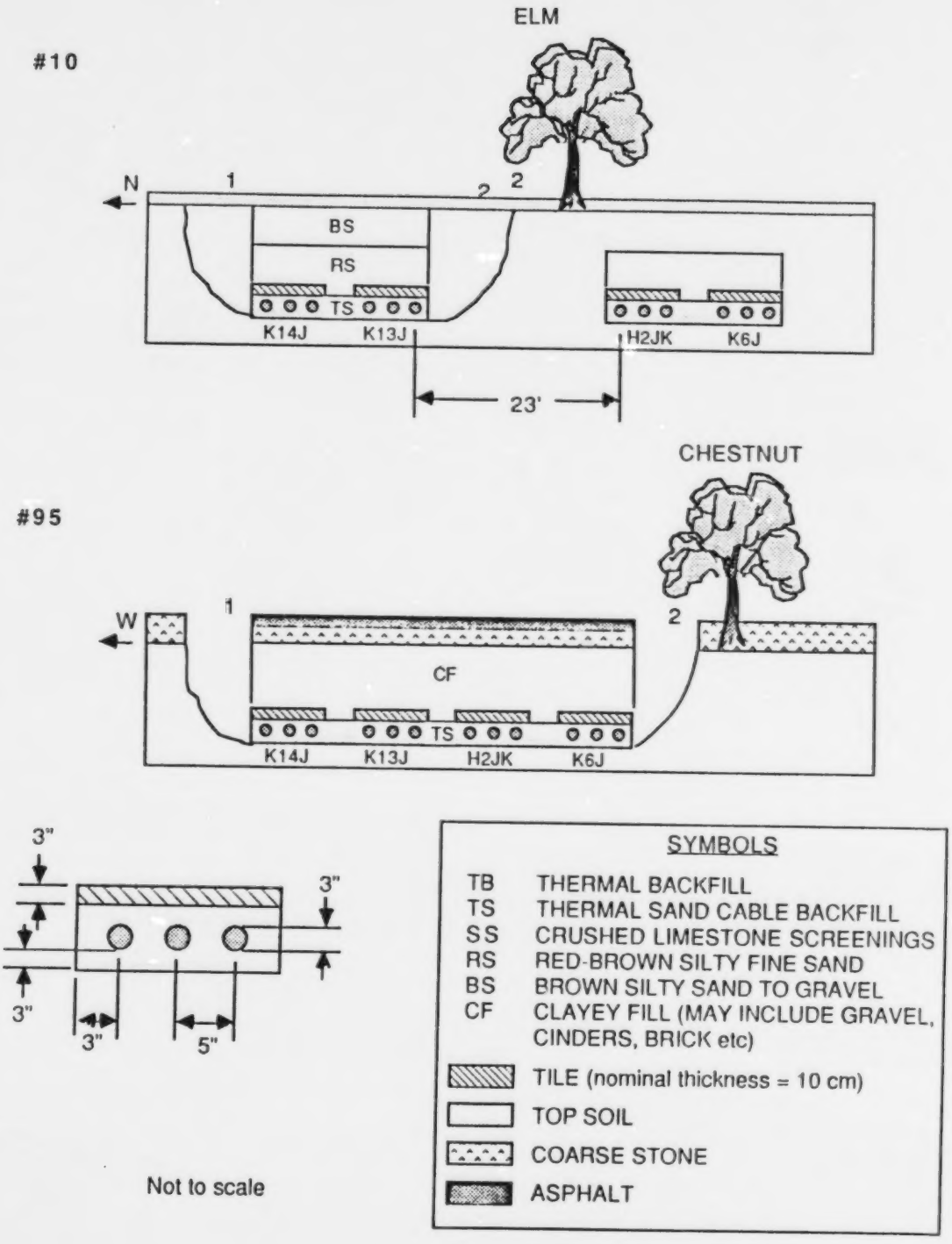


Figure 2. Cable arrangements in the backfill.

The ambient temperature was assumed to be 20°C. Two maximum conductor temperatures were investigated 85°C and 100°C. The results are summarized in Table 2.

TABLE 2

Summary of Steady State Ampacities of Strachan x Riverside Jct Cable Circuits 80% Daily Load Factor

Locations	Near Tree			
	All Circuits in service		1 cct out of service	
	Cond. Temp 85°C	100°C	Cond. Temp. 85°C	100°C
#10	569	633	569	633
#95	566	630	578	642
	No tree			
	749	824	763	840

Under the above conditions the rating of each cable for a steady state condition would be set at 566 A and for emergency conditions at 630 A. The system is designed and operated in such a way that a loss of a single circuit should not cause an overload of the remaining circuits under steady state conditions.

The peak load forecast for Strachan and John stations, translated into current flowing in one cable with one circuit out of service, is given as follows. [12]

Year	1988	1989	1990	1991	1992	1993	1995	1998	2000
Current (A)	719	780	818	808	829	863	930	1034	1113

Comparing the computed cable ampacity with the above values it can be seen that the emergency limits could be exceeded in the summer of 1988. Thus, remedial action might need to be undertaken (ie. this would depend on actual outages) to meet expected load growth. If remedial action is taken (resulting in thermal resistivities as in case without trees), the new circuit will be required by 1992 (assuming 100°C conductor temperature).

In addition to the necessity to simplify the models to apply the IEC method, a constant ambient temperature had to be assumed in the above computations and single values of the load forecast and thermal resistivities were used.

Because of repeated high thermal resistivity measurements, it is realistic to assume that a "worst case" condition is applicable in the case when no remedial action is taken. However, if something is done to mitigate the adverse effect of the trees, the thermal resistivities of the backfill and native soil will follow seasonal variations and taking single values for the loading, temperature and thermal resistivities will lead to unduly pessimistic conclusions.

The following sections describe an application of the finite element method for temperature field computations and application of probabilistic techniques, involving the Monte Carlo simulation, for more accurate analysis of cable system capabilities.

3.0 PROBABILISTIC ANALYSIS

One of the options available to increase ampacities of these cables, is to replace sand backfill in the vicinity of the trees with another material having more favourable thermal properties. The deterministic studies reported above would yield an ampacity of 749 A in steady state and 840 A in emergency conditions. A 10% increase in the ampacity of a transmission circuit in a 3% growth environment, for example, may defer the need for substantial capital expenditures for 3 years. System Planning Division has estimated that the 1987 cost of an underground cable installation in an urban area could be $2.5 million per km. The present value of deferring such a cable installation from the early to the mid 1990's is estimated to be at least $50,000 per km per year of deferment. To confidently determine the higher cable ampacity which would permit such a deferment, reliable advanced thermal analysis techniques are required.

In the past four years Ontario Hydro and IREQ have been working on a joint research project, funded by Canadian Electrical Association (CEA), to develop and test user friendly computer programs for advanced cable ampacity calculations. Computer models using sophisticated numerical techniques, such as finite element methods have been developed in the aforementioned project to achieve improved accuracy. Coupled heat/moisture models have been developed to describe the soil thermal stability phenomenon. This report describes how these powerful tools are used to benefit the design of new cable installations and how they were adapted for use in assessing existing cable systems for the purpose of uprating.

The measurements performed for this project underscored the fact that the soil and ampacity thermal resistivity is one of the most important factors determining cable operating temperature. The value of this parameter depends on the soil type used and the moisture content in the soil. The moisture content, on the other hand, depends to a large extent on weather conditions. Another important factor which determines cable operating temperature is the current flowing in the circuit. This is also a somewhat random quantity which has to be predicted, if the cable operating temperature is to be computed with reasonable accuracy.

Since there are many uncertainties encountered in cable rating, a probabilistic approach seems to be more appropriate than traditional deterministic techniques with the associated safety factors if quantitative assessment of overload risk is needed to minimize cost. A probabilistic approach using the finite element method yields the probability distribution function of the cable temperatures. A cable engineer can compare the distribution with the design limits to obtain uprating margins. This report introduces a probabilistic method for thermal analysis of power cables in the system planning process. The probability distribution of cable operating temperature is used in performing cost-benefit analysis of delaying new cable installations through cable uprating.

4.0 FUNDAMENTAL CONCEPTS IN THE FINITE ELEMENT METHOD

The general equation for heat conduction problem is

$$C dT/dt = \nabla(\lambda \nabla T) + Q \qquad (1)$$

where T (°C) is the temperature, ∇T is the temperature gradient, C ($J/m^3 \cdot °C$) is the heat capacitance, t (s) is the time, λ ($W/°K \cdot m$) is the medium thermal conductivity and Q (W/m^3) is the heat generated within the field. In this equation, the thermal conductivity λ is assumed to be constant. Equation (1) has been solved using the finite element method [2,3,4].

Finite element methods divide the solution region into small, discrete regions called elements. There exist many types of finite elements, but the most common shape for two-dimensional elements is the triangle. In this report, the words "triangle", "element" and "finite element" will be used interchangeably. The vertices of the triangles are called nodal points or nodes. The temperature at each nodal point is denoted by a variable which is to be determined.

Within each element, the solutions are represented by polynomials with unknown coefficients representing the nodal temperatures. The nodal values must now be "adjusted" so that they provide the "best" approximation possible to the true temperature distribution. This adjustment is accomplished by minimizing the functional χ defined below (this is the so-called Rayleigh-Ritz method [5]).

$$\chi = \int_{\text{area}} \frac{1}{2} \lambda \left(\frac{\partial T}{\partial x}\right)^2 + \frac{1}{2} \lambda \left(\frac{\partial T}{\partial y}\right)^2 \, dxdy - \int_C qT \, ds + \int_C \alpha(0.5T_2 - T_a T) \, ds$$

where q is the value of a heat flux at the constant heat flux boundary, α is the convection coefficient and T_a is the ambient temperature at the convective boundary. The functional χ can be interpreted as the total thermal energy of the system. The solution is obtained by finding the function T which minimizes the functional χ. This method is known as a variational principle approach [5].

This approach results in a matrix equation whose solution vector contains coefficients of the approximating polynomials. The answer is then known throughout the solution region. The steady state matrix equation has the form:

$$\mathbf{G \cdot T = R} \qquad (2)$$

and for time dependent computations the equation evaluating temperature at time t is:

$$[G_{t-\Delta t}/3 + C_{t-\Delta t}/(2\Delta t)] \cdot T_t$$

$$= R_{t-\Delta t} - G_{t-\Delta t}(T_{t-\Delta t} - T_{t-2\Delta t})/3 + C_{t-\Delta t}/(2\Delta t) \cdot T_{t-2\Delta t} \qquad (3)$$

where:

- **T** is a 1xn vector of nodal temperatures;
- **G** is a nxn matrix with the elements depending on thermal conductance and convection coefficient (if a convective boundary is represented);
- **C** is a nxn capacitance matrix;
- **R** is a 1xn vector with elements depending on heat generated in the cables and on boundary conditions;
- Δt is a time interval.

5.0 STATISTICAL VARIATIONS OF CABLE CIRCUIT PARAMETERS

For a given cable type and size the temperature T is, in general, a function of the cable loading as well as other soil and ambient parameters which constitute the thermal circuit of the cable. The following parameters exhibit statistical variations:

- ρ_s native soil thermal resistivity (°K•m/W)
- ρ_b backfill thermal resistivity (°K•m/W)
- I cable loading (A)
- T_a ambient temperature (°C).

In conventional techniques, the cable temperature is evaluated on the basis of known or assumed values of cable circuit parameters (base-case or nominal values). In practice, however, these parameters are subject to random operating, geographical and seasonal variations which affect the cable temperatures, hence the allowable loading level of any particular cable. The random parameter variations are described by associated probability distributions which are evaluated on the basis of statistical and historical records. The following sections describe how these distributions are obtained from historical record.

5.1 Load Probability Distribution

The probability distribution of the cable loadings was obtained from the historical records for the circuits under consideration. A histogram showing the number of occurrences of a particular MVA daily peak demand divided by the monthly peak was obtained for the month of August of 1984, 1985 and 1986 and is shown in Figure 3. The month of August was chosen because this is the most critical month from cable operating temperature point of view.

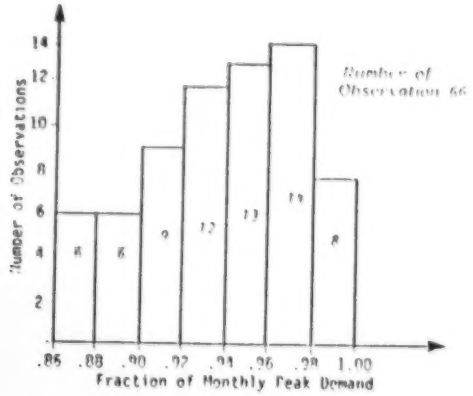


Figure 3. Histogram of the daily peak load for the month of August.

The following peak demands have been estimated for cables supplying Strachan and John stations for some of the years[12]:

Year	1988	1989	1990	1991	1992	1993	1995	1998	2000
Load (MVA)	407	447	468	464	476	492	522	569	601

The finite element programs require specification of the total cable losses as an input variable. The above MW values were used to obtain the conductor current. A computer program using the IEC method [6] was used to estimate the total losses in the cable. Application of this program has shown that conductor losses constitute about 90% of all losses in this particular cable.

5.2 Ambient Temperature Probability Distribution

To obtain the probability density function of the ambient mean temperature, the weather records for downtown Toronto provided by Environment Canada for the period from January 1, 1920 to December 31, 1985 were analyzed. Figure 4 shows the histogram of the mean daily temperature in the month of August based on the 66 years of data and 2046 observations.

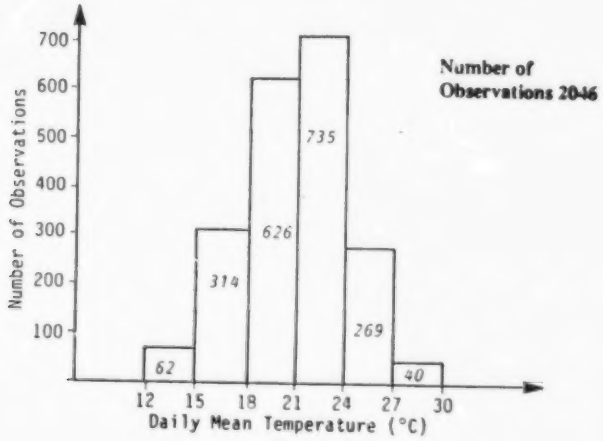


Figure 4. Histogram of the ambient temperature in downtown Toronto for the month of August.

5.3 Native Soil and Backfill Probability Distributions

The thermal resistivity of the thermal backfill and the surrounding native soil are governed by their composition, density and moisture content. For a given backfill/soil type, the moisture content is the most important factor influencing the thermal resistivity. For sand in the Toronto area, the curves relating variation of thermal resistivity with the changes in the soil moisture content were established as shown in Figure 5. In order to determine these relationships, several field tests and laboratory measurements should be performed along the cable route. Alternatively, published relationships between these quantities could be used [7,8].

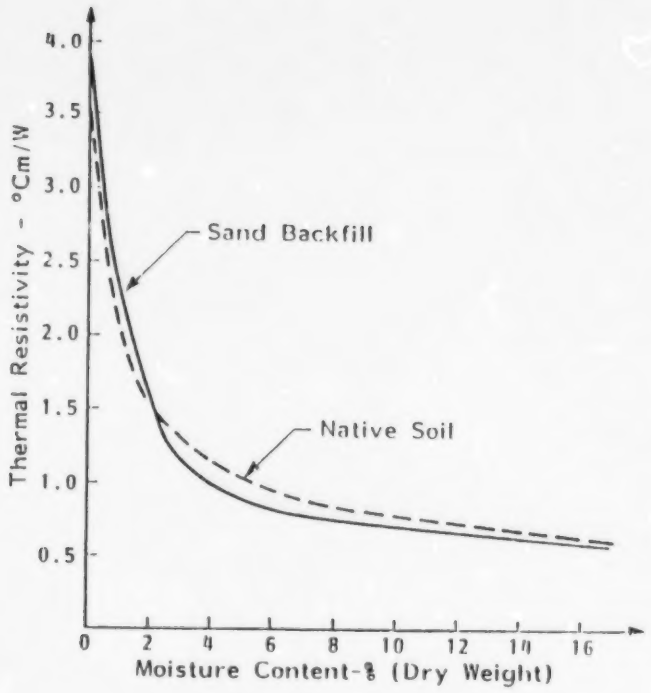


Figure 5. Moisture vs thermal resistivity curves for the material in the vicinity of the cables.

The observed high values of thermal resistivity in the low moisture content range are partially due to the poor compaction of the sand backfill. Poor compaction of the sand leads to decreased density and to high dependence of the thermal resistivity on moisture content.

The probability distribution curves for sand backfill and native soil (ρ_s, ρ_b) were constructed from the statistical analysis of weather data for the region, cable loadings, and the established relationship between the thermal resistivity and soil moisture content. To determine the initial amount of water in the soil, a computer program has been written using the formulas developed by C.W. Thornthwaite [9] for agricultural purposes and adapted to the case of cable backfills in the Toronto area climate. These formulas build a soil-water balance on the basis of initial moisture content in the soil, recorded precipitation, maximum moisture content which the soil can sustain (so called field capacity) and recorded temperatures. A brief description of the method is given in Appendix A.

The weekly precipitation values required in the Thornthwaite formulas were obtained from the 330 records for the month of August provided by Environment Canada for the 66 year period. The application of the Thornthwaite method gave 330 values of the initial moisture content in the soil as shown in Figure 6. Note that not only does the water content vary according to climatic conditions, but also because of the thermal gradients caused by the buried loaded cables. The Thornthwaite formulas deal with the moisture fluctuation in the soil caused by the climatic conditions only.

Current carrying power cables may cause moisture migration away from cables depending on loading and temperature gradients. Therefore, the moisture content of the sand backfill may be somewhat lower than predicted by the Thornthwaite method. In order to take into account the effect of this migration, a series of studies, using the heat/moisture computer program [10], were performed to obtain the actual values of the thermal resistivities of the backfill to be used in the Monte Carlo simulations. The method employed is described in Appendix B.

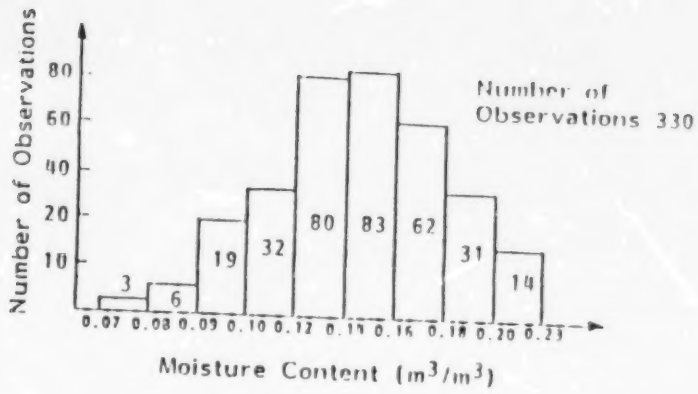


Figure 6. Histogram of the initial moisture content in the soil.

6.0 COMPUTATION OF TEMPERATURE DISTRIBUTION USING MONTE CARLO SIMULATION

6.1 Steady State Analysis

The dependence of temperatures around buried power cables on the input parameters can be generally described by a functional relationship as shown below:

$$T = T(I, \rho_s, \rho_b, T_a, \theta_0, \text{constant parameters}) \quad (3)$$

where θ_0 is the initial moisture content and, both ρ_s and ρ_b are functions of θ_0.

Using the Monte Carlo simulation procedure, the probability density function of T is obtained as follows. A point is selected from each of the input densities (that is the values of I, θ_0, and T_a are selected). The value of ρ_s is read from Figure 5 and the value of ρ_b is calculated taking into account the initial moisture content θ_0, and the selected value of cable loading, I, as described in Appendix B.

The resulting probability densities of the thermal resistivities are shown in Figs. 7 and 8.

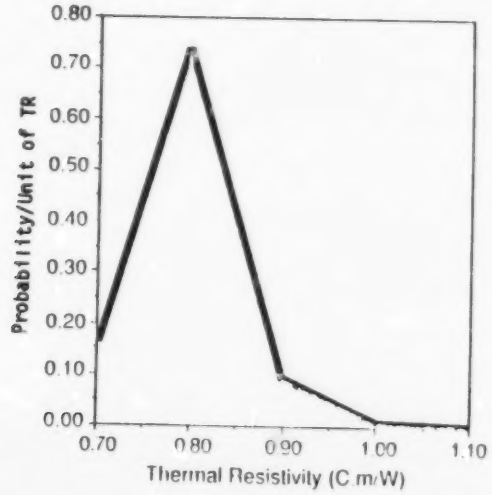


Figure 7. Probability density function of the native soil thermal resistivity.

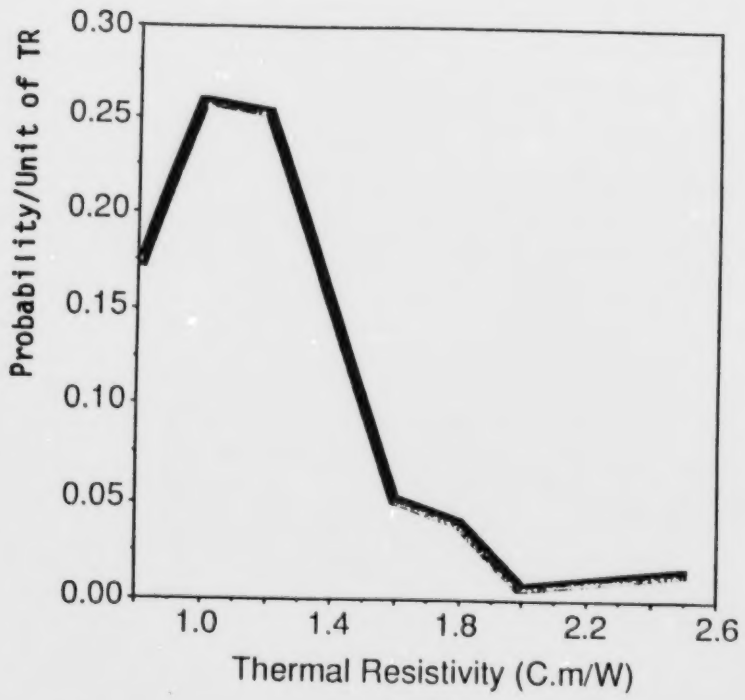


Figure 8. Probability density function of the Backfill Thermal Resistivity (1995).

Next, equation (2) is used to compute resulting temperatures. The selection is performed in such a way that after a sufficient number of selections from a given density function are made and the histogram of the selections is drawn, the resulting shape will closely resemble the input density. The key word in the previous sentence is "sufficient". In practice this means hundreds and even thousands of Monte Carlo simulations have to be made depending on the shapes of the original density functions. In the study reported here, over 10,000 simulations were performed for each of the years studied.

The general computational procedure is shown in Figure 9.

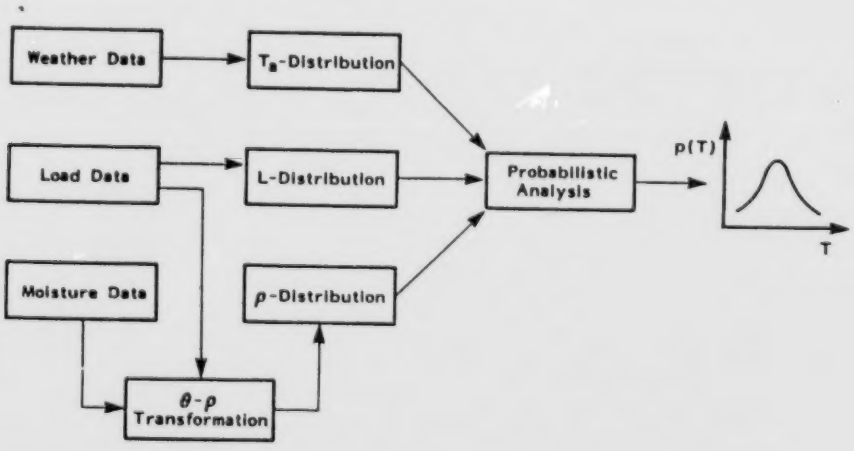


Figure 9. General computational procedure for Monte Carlo analysis of power cables.

6.2 *Transient Analysis*

The procedure described in Figure 9 gives the probability density function for the steady state conditions. For computation of the emergency temperature distribution, the following procedure was adopted.

First, the initial conditions are established following the Monte Carlo selection procedure as described above. The selected values of ρ_s, ρ_b, I and T_a were used to compute the pre-contingency temperature T_0. It is assumed that the initial conditions none lasted long enough for steady state conditions to be reached. Next, a circuit is taken out of service and its load is redistributed among the remaining cables. This results in a new value of heat losses in all the cables. The outage duration is sampled from the appropriate probability density function. To obtain this function, the outage data of circuits under consideration for the period from 1969 to 1986 were analyzed. Only outages longer than 1 hour were considered because of the long time constants of the soil. There were 72 such outages and the resulting histogram of outage durations is shown in Figure 10.

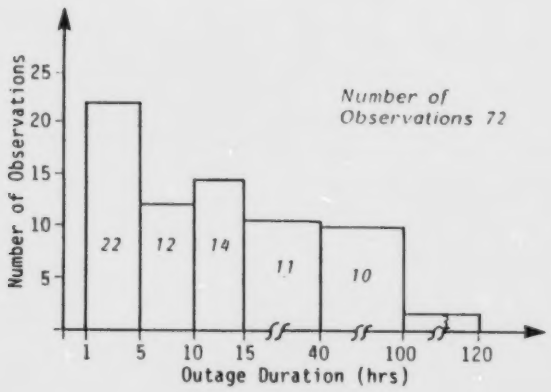


Figure 10. Histogram of outage durations > 1 hr.

In the transient analysis, it is assumed that the loading and environmental parameters remain unchanged for the selected outage duration. This yields pessimistic values of the temperature because the probability density function for the current flowing in the cables corresponds to the peak load at the customer buses. The temperature at the end of the analyzed period is then recorded in the emergency temperature histograms.

In order to be able to perform a large number of simulations, each involving a solution of full steady state and transient equations (2) and (3), an efficient program for an IBM PC compatible computer was developed. The computations are performed very fast mainly due to the underlying assumption of the constant boundary and environmental conditions during the cable emergency loading period. The conductance matrices in equations (2) and (3) are evaluated and factorized only once during each simulation.

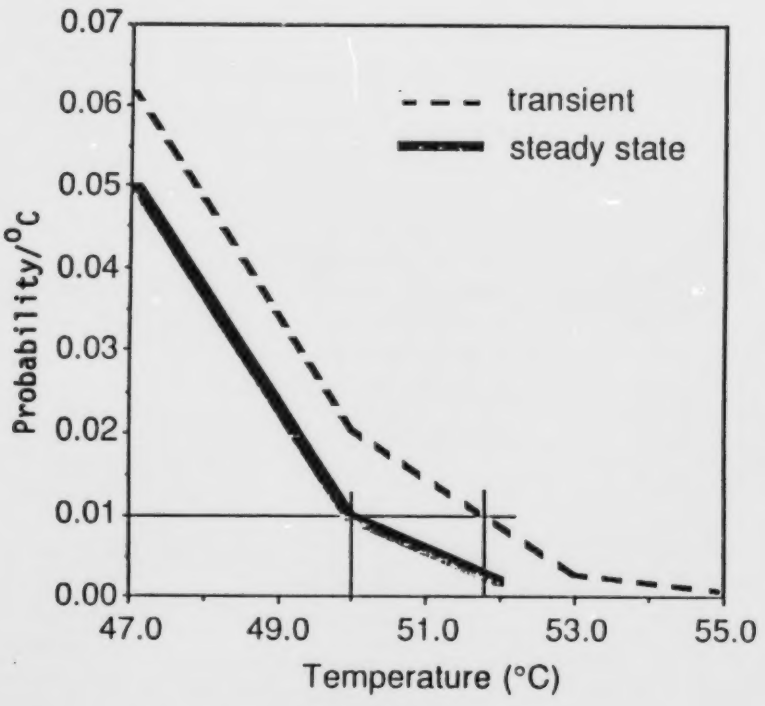
The time step in each iteration has been chosen to be equal to 2 hours. The outages on the cable circuits considered tend to be very long as indicated in the histogram in Figure 10. However, the number of transient iterations is not very large because the new steady state conditions corresponding to the emergency cable loadings are reached fairly quickly. The program takes advantage of this situation.

corresponding to the emergency cable loadings are reached fairly quickly. The program takes advantage of this situation.

Figure 11 shows the tails of the cumulative probability distribution function of the temperature on the external surface of the fifth cable from the left (see Figure 2) based on 1990 loading conditions. This is the hottest cable in the group. Both the steady state and emergency temperatures are shown. The critical cable surface temperature is usually in the range of 50 - 60°C. Assuming the critical value in this study is 60°C for steady state and 65°C for transient conditions, respectively, it can be observed that these temperatures are not expected to be reached in that year.

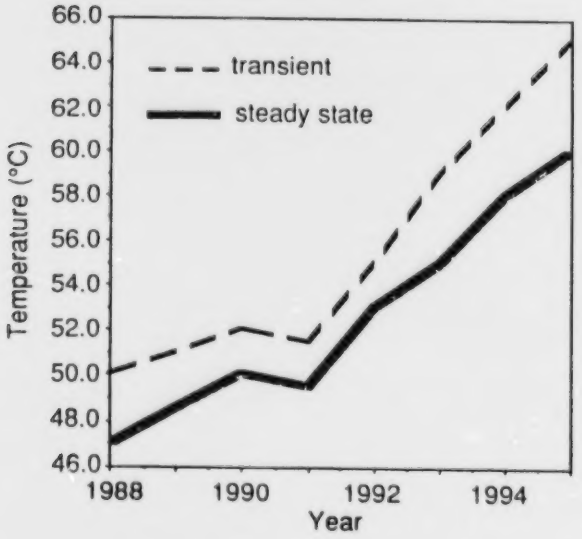
7.0 ECONOMIC ANALYSIS

In order to decide when the capacity of existing circuits is no longer sufficient, an analysis similar to that described above was performed for years 1988, 1989, 1993, etc. Suppose that an acceptable risk "p_{max}" of exceeding cable design temperatures is selected. For the purpose of this discussion the value of $p_{max} = 0.01$ was chosen. The value of p_{max} could be different for normal and emergency conditions. The corresponding steady state and emergency temperatures are then read from probability distribution functions for each year as shown in Figure 11. These values are then plotted as illustrated in Figure 12. From this figure a critical year is selected in which either the steady state or emergency cable operating temperature is exceeded. For this example, a new cable would be recommended for 1995.



Temperature 1990

Figure 11. ***High temperature end of the probability distribution of steady state and emergency cable temperatures.***



Cable Surface Temperature (p=0.01)

Figure 12. Yearly cable surface temperature .

As discussed in Section 2, based on *standard* deterministic criterion, the new cable would have to be installed by 1992 (assuming that a remedial action is taken immediately to modify thermal resistivities in the vicinity of trees). Assume an annual discount rate of 10.5% and that the payments start in 1992. The installed cost of 1 km of new cable in 1987 is estimated at $2,500,000. The expected cost escalation rate is 6.5%. The three year postponement gives the following savings in the new cable installation.

	Installed Cost of Cable per km in year shown ($millions)	Present Value (1987) Factor	Present Value of Expenditure in year shown ($millions)
1992	$1.065^5 \times 2.5 = 3.4252$	0.5493	1.881
1995	$1.065^8 \times 2.5 = 4.1375$	0.4071	1.684
Saving			0.197

The economic value of deferring the installation of 1 km of cable from 1992 to 1995 is $197,000. The economic value of deferring the installation of 5 km of cable would be $985,000.

The year chosen for the installation of new facilities will depend on the critical value p_{max} of acceptable risk selected and the cable maximum operating temperatures permitted. The last quantity is determined by the cable manufacturer and is assumed to be constant. The value of p_{max} would

have to be established for each cable installation separately, and it would depend on company policies and cable operating conditions.

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REFERENCES

[1] "Calculation of the Continuous Current Rating of Cables (100% load factor)", IEC Publication 287, 1982, 2nd edition.

[2] Flatabo, N., "Transient Heat Conduction Problem in Power Cables Solved by the Finite Element Method", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-92, 1973, pp.161-168.

[3] El-Kady, M.A., Chu, F.Y., Radhakrishna, H., Horrocks, D.J., Ganton, R., "A Probabilistic Approach to Power Cable Thermal Analysis and Ampacity Calculation", IEEE Trans. on Power Delivery, Vol. PAS-103, 1984, pp 2735-2740.

[4] Anders, G.J., Bedard, N., Chaaban, M., Ganton, R.W., "New Approach to Ampacity Evaluation of Cables in Ducts Using Finite Element Technique", to be published in the IEEE Trans. on Power Delivery.

[5] Zienkiewicz, O.C., The Finite Element Method in Engineering Science, McGraw-Hill, London 1971.

[6] Anders, G.J., El-Kady, M.A., Horrocks, D.J., Ganton, R.W., Motlis, J., "Power Cable Ampacity Calculations on a Desktop Computer: A Review", IEE Proceedings, Part C, Vol. 133, No.7, November 1986, pp.431-436.

[7] Groeneveld, G.J., Snijders, A.L., Koopmans, G., Vermeer, J., "Improved Method to Calculate the Critical Conditions for Drying Out Sandy Soils Around Power Cables", IEE Proceedings, Vol 131, Pt.C, No.2, March 1984, pp. 42-53.

[8] Abdel-Hadi, O.N., "Flow of Heat and Water Around Underground Power Cables", Ph.D. thesis presented at the University of California at Berkeley, October 1978.

[9] Thornthwaite, C.W., "An Approach Toward a Rational Classification of Climate", Geographical Review, Vol. 38, 1948.

[10] Anders, G.J., Radhakrishna, H.S., "Computation on the Temperature Field and Moisture Content in the Vicinity of Current Carrying Underground Power Cables", to be published in IEE Proceedings, Part C.

[11] Vassallo, D.G., "The Influence of Climatic Conditions on the Moisture Content of Cable Backfills", Ontario Hydro Research Division Report, No. E69-9-K, February 1969.

[12] Krpan, M., "Loaded Forecast for Strachan & John Stations" notes distributed to Task Force Members.

APPENDIX A

Computations of the Mean Moisture Content in Media Surrounding UNderground Power Cables

A1. Introduction

Soils used in underground cable installations are composed of granulated minerals, water, air and organic materials. The thermal resistivities of these components range from 0.11°K•m/W for quartz to 0.165°K•m/W for water and 40.0°K•m/W for air. To obtain the minimum thermal resistivity, the soil should contain the maximum amount of rock particles and the minimum amount of air.

As it is impossible to pack solid particles without interstices, it is better to have them filled with water than with air. Water also acts as a binder helping to compact the soil.

In general, the factors appearing to affect the thermal resistivity of most soils are:

1. soil density;
2. moisture content;
3. the volume of air voids in relation to the volume of water;
4. particle surface area per unit volume; and
5. mineral composition of solids.

Moisture content in particular is not constant but varies according to climatic conditions, and may vary because of the thermal gradients caused by the heat dissipated by buried power cables.

The method of evaluating moisture content in the soil discussed here deals with moisture fluctuations in the soil caused by climatic conditions only.

Formulas developed by Thornthwaite for agricultural purposes are adopted in the case of cable backfills and in the Toronto area climate [9,11].

A2. Soil-Water Balance

Water precipitates in the form of snow, rain or dew. Part of it is lost as surface runoff, part of it percolates through the soil layers and the remainder is retained in the soil. The precipitation fraction which is retained depends on the type of soil and its previous moisture content. The maximum moisture content which a soil can sustain for any extended period is defined as the field capacity, usually expressed as a percentage by weight of dry soil or as "inches of water" (per foot of depth of soil). Water also returns to the atmosphere by evaporation from water ground surfaces and by transpiration from vegetation.

The interest here is limited to evaporation from soil surface and transpiration from plants; these two effects combined are called *evapotranspiration*. There is a distinction between the amount of water that actually evapotranspirates and the amount which would transpire and evaporate if it were available.

If enough water is available evapotranspiration rises to a maximum that depends only on the climate. This may be called *potential evapotranspiration*. Thus, the potential evapotranspiration

does not represent the actual transfer of water to the atmosphere but the *transfer that would take place under ideal conditions of moisture content and vegetation.* Nevertheless, the research reported in [9] and [11] indicate that the results obtained considering the potential evapotranspiration as a basic element are consistent with those obtained with independent investigations.

A3. Determination of Potential Evapotranspiration

In each area the relation between mean weekly temperature and potential evapotranspiration can be expressed by an equation of the form:

$$PE = 0.145\,[50 \cdot (TW-32)/(9 \cdot I)]^{A} \qquad \text{(A1)}$$

where:

PE = weekly potential evapotranspiration (inches of water)
TW = mean temperature for the past week in degrees F
I = heat index for the current year
A = a constant function of A.

The constant I is given as a sum of 12 monthly heat indexes, each given by the equation:

$$i = [(TM-32)/9]^{1.514}$$

where:

TM = mean monthly temperature.

Thus

$$I = \sum_{i=1}^{12} i$$

The relationship between I and A is closely approximated by the expression:

$$A = 675 \cdot 10^{-9} \cdot I^{3} - 771 \cdot 10^{-7} \cdot I^{2} + \cdot 10^{-5} \cdot I + 0.49239$$

Equation (A1) gives unadjusted values of potential evapotranspiration as this equation was first developed for a standard month of 30 days, each day having 12 hours of possible sunshine. It is necessary to apply a correction factor because the number of sunlight hours, when evapotranspiration is higher, varies with the season and latitude, and the month length ranging from 28 to 31 days.

The correction factors C, according to Thornthwaite, for the Toronto area are:

0.81, 0.82, 1.02, 1.12, 1.26, 1.28, 1.29, 1.20, 1.04, 10.95, 0.81 and 0.77

for January to December, respectively. The adjusted potential evapotranspiration, PEC, is now given by:

$$PEC = PE \cdot C.$$

A4. Weekly Soil-Water Balance Calculations

The computations of weekly water balance in the soil start in the month in which frost leaves the ground. It is assumed that at this point the soil is saturated with water, ie. its moisture content is at field capacity. The starting and ending dates in a year for water balance computation have been established by examining the mean soil temperatures provided by Environment Canada for the 66-year period.

The total moisture content in a given week is obtained by summing the soil moisture content of the previous week and weekly total precipitation. The actual evapotranspiration, AE, is assumed to be proportional to potential evapotranspiration, PEC. In fact, when the soil moisture content falls below the field capacity, the rate of evapotranspiration falls too, thus

$$AE = \frac{\text{Previous balance}}{\text{Field capacity}} \cdot PEC$$

The water balance is obtained by subtracting the actual evapotranspiration from the total moisture content.

APPENDIX B

Estimation of Backfill Thermal Resistivity

For the purpose of Monte Carlo analysis discussed in this report, the following procedure has been developed for modification of the thermal resistance of the backfill to take into account the moisture migration problem due to thermal gradients.

In a separate study, the curves shown in Figure B1 have been established for several representative initial moisture conditions (θ_0) in the soil. These curves relate the moisture content, θ, in the soil above the cable to the amount of heat, Q, generated in each cable.

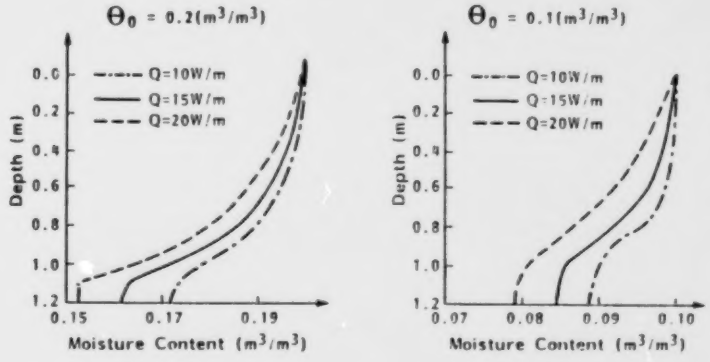

Figure B1. Effect of the heat rate Q on the moisture distribution in the sand.

The information contained in the curves in Figure B1 is used in the analysis in the following way.

At the beginning of each Monte Carlo simulation values of Q and θ_0 are selected at random from the respective histograms. Next, the initial moisture content θ_0 is used to select the appropriate "graph" and the value of Q is used to select the appropriate "curve" in the chosen graph. The moisture content at the cable surface read from this curve is recorded and the thermal resistivity computed from the curves in Figure 5. This resistivity is used for the entire backfill zone. The detailed analysis of the moisture migration in the vicinity of the cables has shown that the low moisture content prevails in the entire backfill zone (see Figure B2) and the moisture content increases quite rapidly above this zone (see Figure B1).

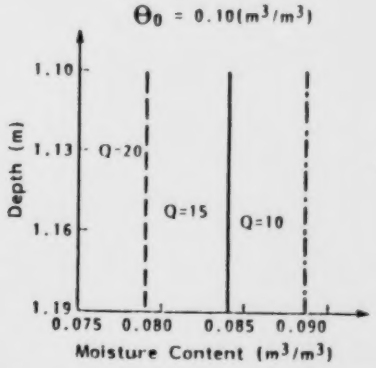

Figure B2. Moisture content profile in the immediate vicinity of the cables.

CREATION OF THE CURVES IN FIGURE B1

One way to create the curves shown in Figure B1 would be to conduct laboratory experiments for the sand under consideration as described, for instance, in [7]. Another possibility is to use available analytical tools to determine the steady state moisture distribution around the cable for different values of Q.

In the example under consideration, the heat loss values, Q, range from 9.6 W/m to 25.6 W/m with all circuits in service and from 15.0 W/m to 33.0 W/m with one circuit out of service. These losses were established for the years 1993-2000 based on the given total peak demand at stations A and B. The curves similar to those shown in Figure B1 were developed for values of losses varying by 1 W/m in the defined range.

Analysis of the results of moisture content computations in downtown Toronto, performed with the aid of the Thornthwaite method for the 66-year period, revealed that the moisture content in the sandy backfill varies between 32 and 95% of field capacity in the month of August. Ten families of curves shown in Figure B1 have been developed for the initial moisture content in the range of 32-95% of field capacity.

The following considerations were taken into account when computer runs were performed to obtain the graphs shown in Figure B1. Since the Thornthwaite method already takes into account the loss of moisture in the soil due to evapotranspiration, the computer results should reflect the migration of the moisture due to the thermal gradients alone. This has been achieved by specifying constant moisture boundaries at the ground surface and at the bottom of the study region (5m) with the moisture values equal to the initial moisture content. Another boundary consideration which should be taken into account is the value of the ambient air and soil temperatures. In the computer runs performed here, these values were kept constant at 20°C and 15°C, respectively. Sensitivity studies performed have shown that the variations in the boundary temperatures have a very small effect on the computed moisture content.